COMMENT ON CARON AND FOX: SPARSE GRAPHS USING EXCHANGEABLE RANDOM MEASURES

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Caron and Fox tout their proposal as

> "the first fully **generative** and **projective** approach to **sparse** graph modelling [...] with a notion of **exchangeability** that is essential for devising our scalable statistical estimation procedure." (p. 12, emphasis added).

In calling theirs the *first* such approach, the authors brush aside prior work of Barabási and Albert (1999), whose model is also generative, projective, and produces sparse graphs. The Barabási–Albert model is not exchangeable, but neither is the authors'. And while the Barabási–Albert model is inadequate for most statistical purposes, the proposed model is not obviously superior, especially with respect to the highlighted criteria above.

Generative. Though amenable to simulation, the obscure connection between Kallenberg's theory of exchangeable CRMs and the manner in which real networks form makes it hard to glean much practical insight from this model. At least Barabási and Albert's preferential attachment mechanism offers a clear explanation for how sparsity and power law might arise in nature. I elicit no such clarity from the Caron–Fox model.

Projective. Projectivity is important for relating observed network to unobserved population, and is therefore crucial in applications for which inferences extend beyond the sample. Without a credible sampling interpretation, however, the statistical salience of projectivity is moot. Here projectivity involves restricting point processes in $\mathbb{R}_+^2$ to bounded rectangles $[0, \alpha]^2$, whose best known interpretation via p-sampling (Veitch and Roy, 2016) seems unnatural for most conceivable networks applications, including those in Section 8.

Exchangeability and Sparsity. A certain nonchalance about whether and how this model actually *models* real networks betrays an attitude that sees sparsity as an end in itself and exchangeability as a means to that end. Even the authors acknowledge that "exchangeability of the point process [...] does not imply exchangeability of the associated adjacency matrix" (p. 3). So why all the fuss about exchangeability if its primary role here is purely utilitarian? To me, the pervasiveness of "exchangeability" throughout the article is but a head fake for unsuspecting statisticians who, unlike many modern machine learners, understand that exchangeability is far more than a computational contrivance.

Final Comment. The Society's Research Section is all too familiar with the Crane–Dempsey edge exchangeable framework, which meets the above four criteria while staying true to its intended application of interaction networks. For lack of space, I refer the reader to Crane and Dempsey (2015, 2016) for further discussion.

References:

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